4.5 - Relating Three Forms of a Quadratic Equation

#3 $x = \frac{-b}{2a}$

Standard Form
$y = ax^2 + bx + c$

factoring

expand & simplify

#1 partial factoring
or
#2 *completing the square*

Factored Form
$y = a(x - s)(x - t)$

find vertex from zeroes

A.O.S.
Sub x into equation

Vertex Form
$y = a(x - h)^2 + k$

Apr 12-2:18 PM

Ex.1 Expand & simplify each equation to obtain the standard form equation.

(a) $y = 2(x + 5)(x - 1)$

$-x+5x$

$= 2(x^2+4x-5)$

$= 2x^2+8x-10$

(b) $y = -0.5(x - 4)^2 + 3$

$= -0.5(x-4)(x-4)+3$

$= -0.5(x^2-8x+16)+3$

$= -0.5x^2+4x\underbrace{-8+3}$

$= \underset{a}{-0.5}x^2+\underset{b}{4}x\ \underset{c}{-5}$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

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Ex.2 Write y = x^2 - 4x + 3 in factored form and vertex form.

$y = x^2 - 4x + 3$

M	A	N
3	-4	-1 -3

$y = (x-1)(x-3)$

OR

$x-1=0$ $\boxed{x=1}$

$x-3=0$ $\boxed{x=3}$

Zeroes: 1 & 3

A.O.S $= \frac{1+3}{2}$

$x = 2$

Sub x=2 into equation

$y = (2)^2 - 4(2) + 3$

$= 4 - 8 + 3$

$= -1$

Vertex $(2, -1)$

$$\boxed{\begin{aligned} y &= (x-2)^2 - 1 \\ y &= (x-1)(x-3) \end{aligned}}$$

Apr 15-10:32 AM

Ex: Determine the vertex, and the vertex form, of
y = x^2 - 12x + 5

M	A	N
5	-12	

$= x^2 - 12x \underbrace{+36 - 36}_{0} + 5$

$= (x-6)^2 - 36 + 5$

$= (x-6)^2 - 31$

$x = \frac{-b}{2a}$

$x = \frac{12}{2}$

$x = 6$

$y = x^2 - 12x + \boxed{36}$

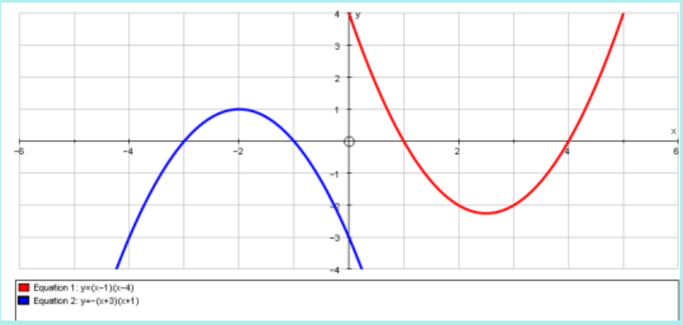
$y = (x-6)^2$

$(x-6)(x-6)$

Apr 15-10:43 AM

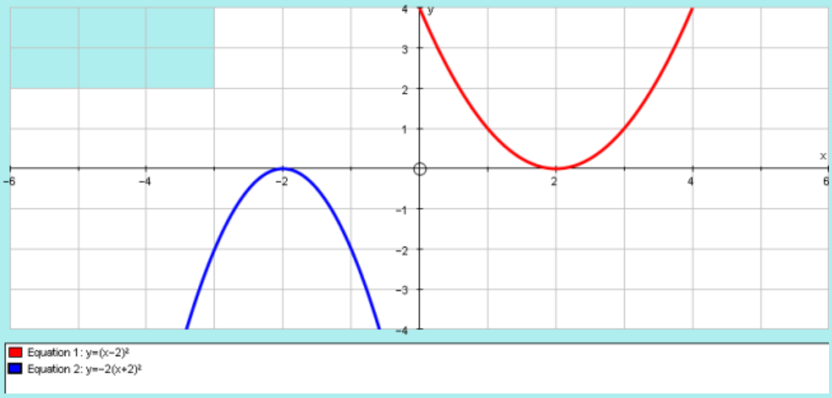
If the parabola crosses the x-axis, the x-coordinates of the crossing points are called the <u>zeroes</u>, or <u>roots</u>, or <u>x-intercepts</u>.

A parabola may have two zeros:



Apr 15-9:06 PM

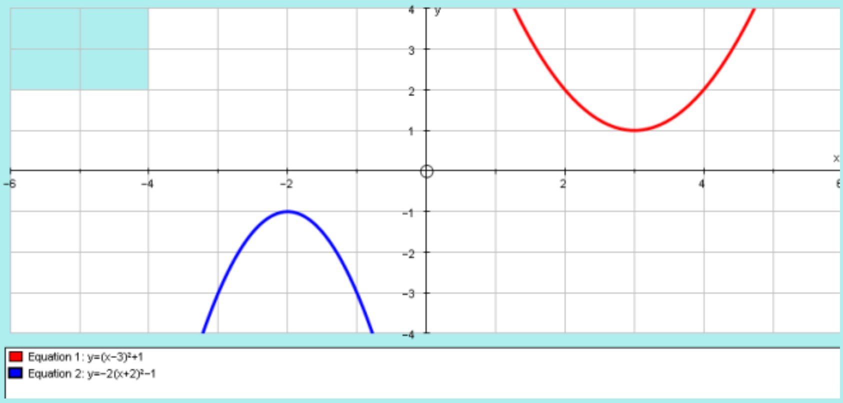
Or one zero:



$(x \pm \#)^2$

Apr 15-9:09 PM

Or no zeroes:

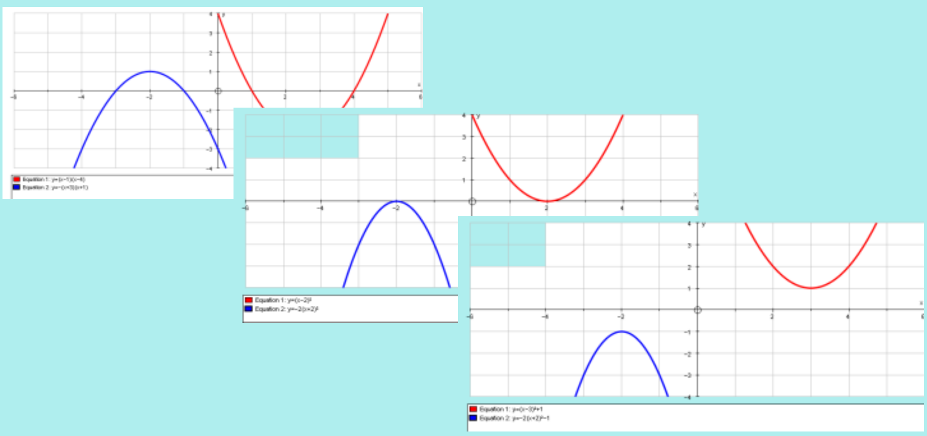


Apr 15-9:12 PM

Recall:

(1) Factored form indicates the <u>zeroes</u> of the quadratic relation.

(2) A quadratic relation can have 2, 1, or no zeroes.



Nov 20-8:17 PM

Not all quadratics have zeroes, which means they cannot be factored. Instead, use symmetry to perform a partial factoring.

1) Determine two points that have the same y-value.
   - start with a point that is given and then find the matching point with the same y-value
   - the y-intercept is usually a good choice

2) Find the x-value of the vertex (h) using symmetry

3) Find the y-value of the vertex (k) by subbing h into the original equation.

Apr 12-2:33 PM

Ex.3 Determine the vertex, and the vertex form, of
$y = x^2 - 12x + 5$

y-intercept (0,5)

by symmetry, there is another point where y=5, sub y=5, solve for x

$y = x^2 - 12x + 5$

$5 = x^2 - 12x + 5$

$0 = x^2 - 12x$

$0 = x(x-12)$

or

$x = 0$ $\quad$ $(0,5)$

$x - 12 = 0$

$x = 12$

$(12,5)$ matching pt

A.O.S $= \frac{0+12}{2}$

$x = 6$

$y = x^2 - 12x + 5$

$y = x^2 - 12x + 36 - 36 + 5$

$y = (x-6)^2 - 31$

Sub x=6 into equation

$y = x^2 - 12x + 5$

$y = (6)^2 - 12(6) + 5$

$= -31$

$\boxed{y = (x-6)^2 - 31}$

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Ex. 4 Determine the vertex, and the vertex form, of
$y = -3x^2 + 15x + 2$

$= -3(x^2 - 5x) + 2$

$= -3(x^2 - 5x + 6.25 - 6.25) + 2$

$= -3[(x - 2.5)^2 - 6.25] + 2$

$= -3(x - 2.5)^2 + 18.75 + 2$

$= -3(x - 2.5)^2 + 20.75$

$(x - 2.5)(x - 2.5)$

Completing the square!!

$y = -3x^2 + 15x + 2$ — y-intercept $(0, 2)$

$2 = -3x^2 + 15x + 2$

$0 = -3x^2 + 15x$

$0 = -3x(x - 5)$

$-3x = 0$, $\boxed{x = 0}$, $(0, 2)$

$x - 5 = 0$, $\boxed{x = 5}$, $(5, 2)$

$A.O.S = \frac{0+5}{2}$

$x = 2.5$

Sub x=2.5 into equation

$= -3x^2 + 15x + 2$

$= -3(2.5)^2 + 15(2.5) + 2$

$= 20.75$

Partial Factoring

Apr 12-2:43 PM

Assigned Work:

p.293 # 4c, 5a, 6a, 9ac, 10c
p.301 # 4c, 5ae, 7c

Apr 15-12:08 PM